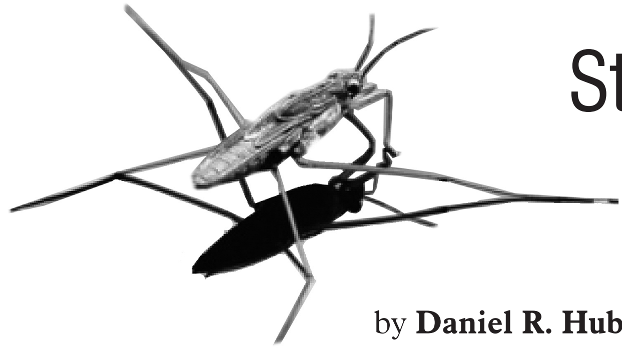

Life on the Edge: Water Striders Use Geometry to Master Physics

by **Daniel R. Huber** and **Leslie B. Jones**

Introduction

Mathematics is a fundamental tool used to explore and understand the world around us. Despite its explanatory power, which elucidates the most complex of relationships, the relevance of mathematics is often missing from mathematics education. As we seek to bring context and purpose to the classroom, connecting our students to intriguing, relevant examples of mathematics in action, the shared interests of biology, geometry, and algebra emerge as natural partners. Geometric variability in biological structures determines the ecological performance (i.e. resource acquisition) of individuals possessing those structures, all of which is described by algebraic expressions. In this article we examine the issue of biological scaling and the manner in which geometric relationships within organisms influence how and where they exist, and the manner in which they evolve.

Biological scaling is the study of size-related effects on the structure and function of organisms (Willmer *et al.*, 2005). As simple as it may sound, increasing in size causes significant changes in the form, function, and physiology of organisms due to the different rates at which linear (x), area (x^2), and volume (x^3) functions change. Among the most important of scaling relationships is the surface area-to-volume ratio of individual cells, the basic units of life (Raven *et al.*, 2014). The surface of a cell is the area over which it exchanges materials with the environment (resources in, wastes out), whereas its internal volume is the physical space in which those materials are consumed and produced. Because volume, a cubic function, grows at a faster rate than area, a square function, larger cells have lower surface area-to-volume ratios, and therefore operate at a physiological deficit; they have relatively less surface area over which to obtain and dispose of resources and wastes, and a relatively larger volume in which those materials are consumed and produced. For aerobic organisms (those which rely upon oxygen for respiration), this surface area-to-volume issue constrains individual cells to be no more than 1 mm in diameter, as those larger than 1 mm cannot distribute oxygen quickly enough to sustain the metabolic needs of the cell, leading to an 'anoxic core' within the cell (Willmer *et al.*, 2005). Moving this relationship from the microscopic to the macroscopic, have you ever wondered why kids are more resilient to falling than adults? The surface area-to-volume ratio strikes again! Adults have relatively more volume (and therefore mass) than kids, and relatively less cross-sectional area to their bones. This unfortunate pairing results in greater bone stress (body weight/bone cross-sectional area) and greater likelihood of failure in older people. We explore a similar relationship between geometry and biology, with an extension to physics, in the intriguing case of water striders, insects that spend their entire lives on the edge of water and air (Fig. 1). In addition to engaging students in an exploration of the biological significance of scaling, this lesson also addresses learning objectives related to perimeter and volume formulas, equation manipulation, and units of measure.

Water Striders

Of the approximately 1700 species of water striders (insects of the family *Gerridae*), 90% live on freshwater and 10% on salt water (Lancaster and Briers, 2008). They have three pairs of legs, the first of which is engaged in communication by making and detecting vibrations at the water's surface, whereas the second and third pairs of legs are for propulsion and steering respectively (Williams and Feltmate, 1992).

Fig. 1 Water strider (*Gerris* spp.)

The ability of a water strider to exist at the water's surface is based on the balance of forces between its body weight and the surface tension force exerted by water. The ratio of these forces, known as the *Baudoin Number* (*Ba*), has been pivotal in the evolution of this enigmatic group of insects. Body weight is the product of body mass (*m*) and gravitational acceleration (*g*), whereas the surface tension force is the product of the surface tension of water (σ) and the contact perimeter over which that force acts (*P*) (Baudoin, 1955; Hu *et al*., 2003).

$$Ba = \frac{body\ weight}{surface\ tension\ force} = \frac{mg}{\sigma P}.$$

The only way to remain at the water's surface is to have a *Baudoin Number Ba* ≤ 1.0, which occurs in insects such as water striders (Hu *et al*., 2003). Mass is the product of volume and density ($m = Vd$), allowing the Baudoin number to be expressed as:

$$Ba = \frac{Vdg}{\sigma P}.$$

As such, the balancing act employed by water striders is dependent on their geometry, which determines both their body volume and the contact perimeter between their legs and the water they so precariously live upon. Body mass is a volume function and increases at a faster rate than body area or perimeter. Therefore, geometry will ultimately limit the size that water striders can attain before crossing the critical threshold when the *Baudoin Number* exceeds 1.0, causing them to sink through the water's surface.

The key inequality $\frac{mg}{\sigma P} < 1$, which can be written as $\frac{Vdg}{\sigma P} < 1$, is equivalent to the following: $\frac{V}{P} < \frac{\sigma}{dg}$.

Given the constants of gravitational acceleration (0.0098 mm/ms^2; Walker, 2009) and the surface tension of water (0.000072 N/mm in freshwater at 25°C; Vargaftik *et al*., 1983), as well as the density of the water strider approximated from scientific literature (0.0012 g/mm^3; Vincent and Wegst, 2004), the relationship between the volume of the water strider's body and its contact perimeter with the water's surface can be further examined:

$$\frac{V}{P} < \frac{0.000072}{0.0012 \times 0.0098} \approx 6.1225.$$

A geometric model of a water strider of average size was developed using a combination of standard geometries (hemisphere, cone, and cylinders) based on measurements taken from scientific literature (Fairbairn, 2005).

The volume of this water strider is approximately 7.6086 mm^3, as can be determined from the given measurements (note that the volume of the legs on both sides of the body must be accounted for). The contact perimeter between the water strider's legs and the water's surface is approximately 24.4800 mm. This perimeter is the equivalent of that of a rectangle passing through the cylinders comprising the tarsal segments of the water strider's leg (segment 3), as these are the only segments in contact with the water. The length of the tarsal segments is given and the diameter of these segments can be determined from their radii (note again that the legs on both sides of the body must be accounted for). The volume to contact perimeter ratio for this water strider is $\frac{7.6086}{24.4800} \approx 0.3108\ \text{mm}^2$, which is well below the threshold of 6.1225 mm^2.

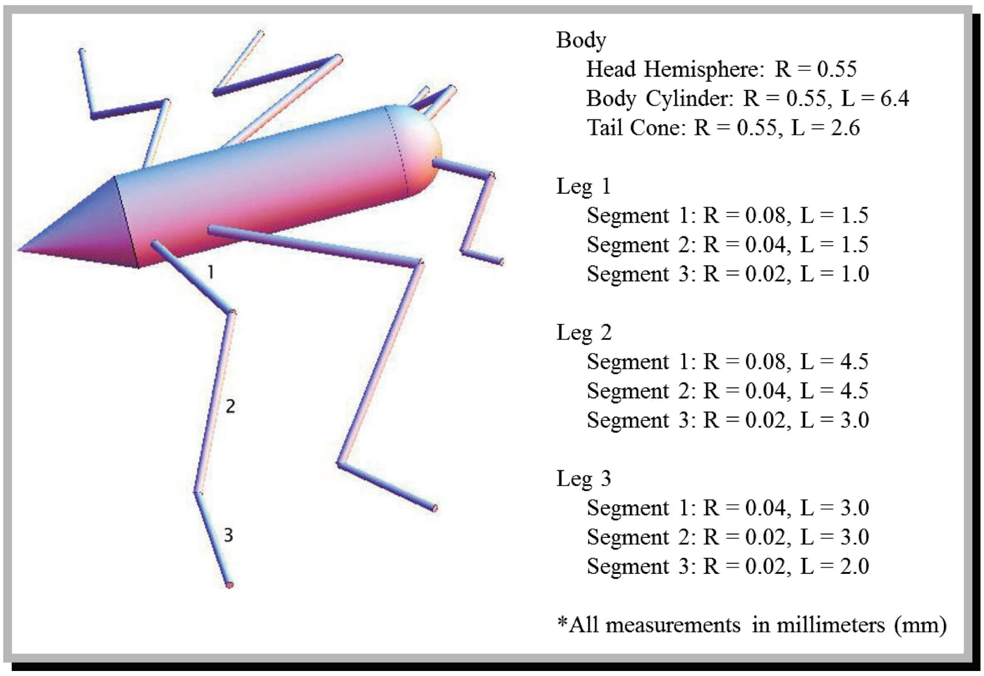

Fig. 2 Geometric model of a generalized water strider with dimensions of all body segments

Assuming isometric growth, wherein the dimensions of a body increase in direct proportion to each other (e.g. two 30/60/90 triangles that differ in their side lengths), let's consider the consequences of doubling each dimension of the water strider's body from our example, and recalculating the volume-to-contact perimeter ratio. Isometric growth is the default assumption for studies of biological scaling, as it entails a consistent pattern of development which has not been acted upon by factors such as natural selection. Algebraically, this doubling of dimensions has the effect of multiplying the original volume by 2^3, and the original perimeter by 2. To take an example, the head of the water strider is modelled by a hemisphere ($V = \frac{2}{3}\pi r^3$). Doubling has the effect of changing the formula to

$$V = \frac{2}{3}\pi(2r)^3 = (2)^3\left(\frac{2}{3}\pi r^3\right).$$

Likewise, both the radius and height of the cylinder representing the body are doubled such that $V = \pi r^2 h$ becomes $V = \pi(2r)^2(2h) = (2)^3\pi r^2 h$.

Adding these two parts together gives us

$$(2)^3\left(\frac{2}{3}\pi r^3\right) + (2)^3\pi r^2 h = (2)^3\left(\frac{2}{3}\pi r^3 + \pi r^2 h\right).$$

Continuing this same process to include the tail and leg volumes gives us a new total volume of the water strider equal to 2^3 multiplied by the original total volume. The contact perimeter, on the other hand, is simply multiplied by a factor of 2, as it is one-dimensional. The new ratio is

$$\frac{\left(2^3\right)7.6086}{(2)24.4800} \approx \left(2^2\right)0.3108\ \text{mm}^2 \approx 1.2432\ \text{mm}^2.$$

Doubling the size yet again (i.e. increasing the original size of the water strider by a factor of four) results in a ratio of

$$\frac{\left(4^3\right)7.6086}{(4)24.4800} \approx \left(4^2\right)0.3108\ \text{mm}^2 \approx 4.9728\ \text{mm}^2.$$

This value is close to our threshold volume-to-contact perimeter ratio of 6.1225 mm^2, suggesting that a quadrupling of size is approximately as large as our water strider can get. In fact, the actual size limit can be found solving for the size multiplier such that

$$\frac{\left(z^3\right)7.6086}{(z)24.4800} \approx 6.1225.$$

Solving for z determines that the maximum size a water strider is capable of attaining is 4.43831 times larger than the original water strider in our example. Incidentally, this is approximately the size of the world's largest water strider *Gigantometra gigas*, which employs the additional trick of allometric (disproportionate) growth of its legs such that they have a larger contact perimeter than would be expected based on the size of its body (Tseng and Rowe, 1999). Thus, it is apparent that the geometry of biology dictates how large and the manner in which water striders can grow, and how far they can push the limits of physics!

In conclusion, we comment that providing context and meaning for mathematical concepts draws students into the learning process. Biological scaling is but one example of the fundamental role mathematics plays in the structure of nature. For more integrated lessons, visit our website:

http://utweb.ut.edu/rwaggett/science-math-master.html.

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