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Why Are Some Problems Easy? New Insights into the Tower of Hanoi

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Introduction

Researchers have found large differences in difficulty and varying amounts of transfer among isomorphs of the Tower of Hanoi (Kotovsky, Hayes, & Simon, 1985; Hayes & Simon, 1977). Because the tasks have the same formal structure, these differences must result from the surface representations. To explain these findings, Kotovsky, et. al. pointed towards the ability to relate the rules to real-world knowledge while Zhang (1997) emphasized the externalization of rules (rules embedded in the external problem representation).

Despite this research, many questions remain about the processes underlying problem solving and transfer of learning. This experiment uses stan– dardized presentations of the isomorphs and presents more problems per participant than in past experiments. These manipulations should enhance transfer and help identify differential difficulty.

Method

Participants were presented with 12 problems for each of three isomorphs of the Tower of Hanoi (the Standard Tower of Hanoi, Monster Move, and Paint Stripping) and two filler tasks. For each task, participants were presented with a description, a set of rules, and an explanation of the interface before beginning. They were instructed to solve each problem by reaching the goal presented on the screen. After finishing the experiment, participants were asked questions to determine how noticeable the relationships among the isomorphs were.

Results and Discussion

The verbal reports were used to determine what information may have transferred from the source isomorph to the target. While some participants claimed to notice a similarity, only 2 (of 37) were able to accurately describe it. Despite this lack of awareness, transfer of learning was clearly shown. Time to solve decreased across isomorph position, $p<.01$ (Figure 1). Also, any of the isomorphs was sufficient to produce transfer. In addition, the degree of transfer was much greater than has been found previously, owing to the standardized interface as well as increased practice. Performance on the Tower of Hanoi was not facilitated by previous exposure to another isomorph (Figure 1; likely due to a floor effect). These findings, combined with the lack of awareness about the similarities, suggest that more general procedural knowledge (execution of general strategies) largely responsible for the transfer. They also suggest that the Tower of Hanoi was relatively easy for participants to solve.

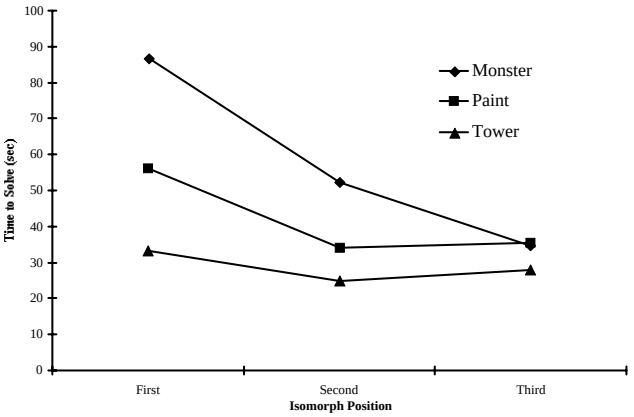

Figure 1. Average time (sec.) to solve problems for each of the isomorphs for each isomorph position.

The comparison of isomorphs showed that the Monster Move isomorph was most difficult, followed by the Paint Stripping isomorph, and finally the standard Tower of Hanoi, $p<.001$. Representational influences seem to drive this effect, with the rules for the Tower of Hanoi being largely inferable from the presentation. In contrast, the Monster Move rules need to be learned explicitly, while at least some of the Paint Stripping rules are not intuitive based on the presentation. These results suggest that the incorporation of problem constraints (rules) into the problem representation can reduce problem difficulty by reducing cognitive load. These results can be generalized beyond the simple problems used here, and suggest simple ways of achieving improved performance in virtually any task domain.

References

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