Math and Nature

* The dawn of the scientific age coincided with the realization that nature can be described mathematically. As stated by Galileo in 1623, the entire universe is written in the language of mathematics. Since this time, quantitative analysis of natural phenomena has been at the heart of scientific inquiry. Owing to this relationship between natural phenomena and its description via mathematics, nature provides a tangible context for math that can be used to invite students into math instruction. Establishing context is essential for developing student interest and comprehension of a topic, and for promoting long-term problem solving skills.

The Importance of Context

* In writing, context is the setting in which a story occurs. In many ways the setting gives meaning to the details of the story and is therefore essential to the development of the plot. As educators we are, in a sense, storytellers that can use context to invite students into the material of our classes. Because of this, we believe that many of the components of good story-telling can be used to improve student engagement and knowledge in the classroom.
  + A lack of context is one of the more common complaints about math education because students are always wondering why a particular mathematical topic is relevant. Context establishes relevance, thereby increasing student interest.
* Pedagogical research has shown that creating context allows students to relate current material to their prior experiences, thereby facilitating experiential and associative learning. This was described by Merrill (2002):
  + First Principles of Instruction:
    - The demonstration principle: Learning is promoted when learners observe a demonstration.
    - The activation principle: Learning is promoted when learners activate prior knowledge or experience.
    - The application principle: Learning is promoted when learners apply the new knowledge.
    - The task-centered principle: Learning is promoted when learners engage in a task-centered instructional strategy.
    - The integration principle: Learning is promoted when learners integrate their new knowledge into their everyday world.
  + The demonstration of new knowledge within a relevant context activates students’ prior knowledge of the subject, which engages them in the material at hand. Once engaged, students are given the opportunity to apply this new knowledge in a task-centered (problem-solving) manner. The association of new knowledge with prior knowledge reinforces the material in memory, thereby allowing learned concepts to be applied to new experiences. Integration of knowledge into new experiences is the essence of problem-solving.

Geometry and Biology

* The use of nature as a context for math is extremely evident with regard to biology and geometry. Biological structures vary greatly in their geometry. Given that differences in the geometry of biological structures affect the function of those structures, biology provides a mechanism for geometric problem solving and for illustrating the consequences of geometric variability.
  + The consequences of geometric variability establish the functional aspect of biology, which is what really invites students into the problem. Given the extensive diversity of organisms, their function can provide context for most any math topic.

Life in Fluids

* Water and air have very different physical properties, which greatly affect the diversity of life that exists in these fluids.

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| --- | --- | --- |
| **Physical Property** | **Water** | **Air** |
| Density (kg/m3) | 997.6 | 1.2 |
| Buoyancy | Yes | No |
| Specific Heat (kJ/kg °C) | 431 | 1.0 |
| Speed of sound (m/s) | 1485 | 343 |

* + The appreciable differences in the density of air and water have fundamental consequences for the movement systems of aquatic and terrestrial organisms. Water’s density is over 800-fold greater than that of air, resulting in the buoyant force that partially supports the weight of aquatic organisms. Not only do aquatic organisms not have to support the entirety of their body weight, but they are also able to push off the dense fluid in which they live as a means of propulsion. Terrestrial organisms get neither of these perks; air does not support any of their body weight and propulsive forces must be applied to the ground for movement to occur. The differences between air and water therefore resulted in major changes to musculoskeletal systems as animals evolved and moved from water onto land. Among vertebrates this change was associated with a shift from lateral undulation of the spine for locomotion to alternating movements of the fore- and hind-limbs.
  + The greater specific heat of water causes it to resist temperature change much more so than air. As a result, aquatic organisms typically go through small, gradual temperature changes over the course of a day, season, year, etc. On the other hand, all terrestrial animals other than birds and mammals (who regulate their internal temperature independently of the external environment) experience dramatic temperature changes as air heats up and cools down over the course of a day, season, year, etc. Temperature has a great effect on organismal function. Therefore, the function of terrestrial organisms is very constrained by air temperature. For example, desert reptiles must bask in the sun each morning to heat their bodies to the point at which they can resume normal muscle function, at which time they resume normal behaviors such as searching for food and mates. The quicker they can heat up the better because time spent basking in the sun is time spent advertising your location to predators.
  + Terrestrial organisms have much better developed senses of hearing than aquatic organisms due, in part, to the much slower speed of sound in air than in water (4.3 times slower). The slowness of sound in air allows enough of a time delay between the arrival of sound waves at either side of your head to permit directional hearing, the ability to determine the direction from which a sound came. Sound travels so quickly in water that it effectively reaches both sides of an aquatic organism’s head at the same time, thereby making directional hearing much more difficult.
* The fluid in which an organism lives greatly affects its structure and function. Not surprisingly, a handful of organisms have evolved the ability to take advantage of the physical properties of both water and air.
  + Porpoising behavior
    - Animals such as dolphins and flying fish are able to use water to generate powerful locomotion forces and then propel themselves into the air where the decreased fluid density allows them to travel further than they would in water.
  + Running on water
    - Basilisk lizards are able to run across water by slapping their large feet vertically down on the water’s surface. They must run, not walk, because the forces created by slapping their feet down on the water only last long enough to briefly support their body weight.
  + Living on water
    - Water striders live completely on the surface of water. Of the approximately 1700 species, 90% live on freshwater and 10% on salt water. They have three pairs of legs, the first of which make and detect vibrations at the water’s surface as a form of communication. The second and third pairs of legs are for propulsion and steering respectively.
      * As though this weren’t a hard enough task, females are doubly challenged because after mating males jump on females’ backs to defend them from other males. This awkward arrangement can last from hours to days.
      * Interestingly, water striders are an active area of robotics research, with teams of engineers around the water seeking to create devices that move on the water’s surface while barely disrupting it.
  + Balance of forces
    - The ability to exist at the water’s surface is based on the balance between an organism’s body weight and the surface tension force exerted by the water. Body weight is the product of body mass (m) and gravitational acceleration (g), whereas the surface tension force is the product of the surface tension of water (σ) and the contact perimeter over which that force acts (P).
    - If this ratio is greater than 1.0 the animal will sink through the water’s surface. The basilisk lizard has a ratio much greater than 1.0 but is able to run (not walk) across the surface because of how rapid its foot movements are. The only way to remain at the water’s surface is to have a ratio less than 1.0, which occurs in insects such as water striders.

Water Strider Geometry

* A geometric model of the water strider has been developed using a combination of standard geometries (hemisphere, cone, and cylinders). The purpose of this exercise is to use the data provided in the model and the properties of standard geometries to determine:
  + The balance of forces (body weight vs. surface tension force) acting on the body of the average water strider.
  + The maximum size at which a water strider can stay afloat.
* Balance of forces acting on the body of the average water strider
  + To determine the balance of forces acting on the body of the average water strider you must first determine the contact perimeter (P) of the leg segments in contact with the water. The contact perimeter determines the magnitude of the surface tension force acting on the water strider. Only the tarsal segments (segment 3) of each leg are in contact with the water. Therefore, the following equation can be used to determine the total contact perimeter for the third segments of the legs on the right and left sides of the body. The width of the leg cylinders is negligible compared to their length and can be ignored.
  + Once the contact perimeter has been determined, the dimensions of the body segments, density of the water strider’s body (0.0012 g/mm3), and the equation for density can be used to determine the water strider’s body mass.
  + Once values have been determined for all variables the balance of forces acting on the water strider’s body can be solved using the equation:
* Maximum size at which a water strider can stay afloat
  + The solution to this problem is determined by investigating the effects of changes in linear dimensions of the properties of standard geometries. In relation to biology, the effect of size-related changes on organismal form and function is known as scaling.
  + The balance of forces acting on the body of the water strider is to be determined after the linear dimensions of the body segments have been increased by 2X, 4X, and 8X. Doing so will indicate the size change at which the surface tension force of the water is no longer able to support the body weight of water strider (MC > 1.0).
  + In arriving at this solution, students should become aware of the fact that lengths, areas, and volumes increase at different rates because:
  + Body weight is a function of volume (our mass exists throughout our length, width, and height). Therefore, body weight increases more rapidly than the ability of the legs to support that weight on the surface of the water and scaling ultimately limits the maximum size that a water strider can be.